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# Forecasting performance of a two-country DSGE model of the Euro area and the United States: the merits of diverging interest-rate rules

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## Abstract

In this paper we estimate and forecast with a small-scale DSGE model of the Euro area and the United States characterized by diverging interest-rate rules using quarterly data from 1996Q2 to 2011Q2. These diverging rules reflect the differing mandates of the ECB and the Fed, respectively. Due to its primary objective of price stability, the ECB is supposed to conduct monetary policy by considering producer-price inflation only, whereas the Fed is assumed to conduct its policy by taking into account the output gap in addition to producer-price inflation (dual mandate). In terms of the RMSE and the MAE, the DSGE model with diverging interest-rate rules outperforms a DSGE model with identical interest-rate rules in almost 70% of all cases for almost all variables across forecast horizons out of sample. It also compares well with BVAR benchmarks. For shorter horizons we find some statistically significant differences in forecasting accuracy between rival models. For forecast horizons three and four, the null hypothesis of equal forecasting accuracy can seldom be rejected.

*Keywords:* DSGE models, Evaluating forecasts, Macroeconomic forecasting, Monetary policy, Open economy macroeconomics, Time series benchmarks.

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# 1 Introduction

The European Central Bank (ECB), as responsible body for conducting monetary policy in the Euro area on the one hand, and the Federal Reserve System (Fed), as its counterpart in the U.S. on the other, are characterized by differing legal mandates. Article 127 of the Treaty on the Functioning of the European Union reads (EU , 2010a):

*“The primary objective of the European System of Central Banks (hereinafter referred to as ‘the ESCB’) shall be to maintain price stability. Without prejudice to the objective of price stability, the ESCB shall support the general economic policies in the Union with a view to contributing to the achievement of the objectives of the Union as laid down in Article 3 of the Treaty on European Union. The ESCB shall act in accordance with the principle of an open market economy with free competition, favouring an efficient allocation of resources, and in compliance with the principles set out in Article 119.”*

In consequence, all other policy objectives of the EU, such as balanced economic growth, a highly competitive social market economy, or full employment and social progress (see EU , 2010b) besides price stability, only play a secondary role in monetary policy.

Concerning the Fed, policy objectives other than price stability, which basically amount to the adaption to the real economy, play a key role as can be seen from Section 2A of the Federal Reserve Act (US , 2000):

*“The Board of Governors of the Federal Reserve System and the Federal Open Market Committee shall maintain long run growth of the monetary and credit aggregates commensurate with the economy’s long run potential to increase production, so as to promote effectively the goals of maximum employment, stable prices, and moderate long-term interest rates.”*

This duality of policy objectives – the so-called dual mandate – may thus create a trade-off between stabilization of inflation and the real economy on the part of the Fed, but should not do so on the part of the ECB.

The purpose of the present study is to investigate if allowing for these institutional differences within a two-country dynamic stochastic general equilibrium (DSGE) framework results in superior forecasting performance relative to [1] a two-country DSGE model *not* taking into account these differences as well as to [2] adequate open-economy time-series benchmarks. Contrary to Smets and Wouters (2005) and Sahuc and Smets (2008), who compare macroeconomic shocks and frictions in the U.S. and Euro area business cycles and investigate differences in the interest-rate policies of the ECB and the Fed, respectively, each with the help of two separately estimated medium-scale DSGE models, we contribute to the literature by performing the analysis within a two-country framework.

The use of a two-country model of the Euro area and the U.S. instead of separate closed-economy models is beneficial for two main reasons. First, Adolfson et al. (2008) find evidence for the Euro area that using an open-economy DSGE model generally improves the forecasting accuracy for key macroeconomic variables

relative to a closed-economy version of their model. Second, the degree of openness between the Euro area and the U.S. as implied by the two-country DSGE framework under scrutiny corresponds to the actual degree of economic interdependence of these two economies (see Section 2.4 for more details).

Authors of articles on two-country DSGE models addressing the issue of optimal monetary policy under discretion or under commitment in its various facets (see, e.g., Clarida *et al.* , 2002; Pappa , 2004; Benigno and Benigno , 2006; Engel , 2011) usually encounter a trade-off between minimizing the volatility of inflation and the volatility of the output gap on the part of both countries' central banks since they typically assume identical mandates for both central banks.

Our idea, however, is to express the differences in mandates and policy objectives of the two real-world central banks under scrutiny – the ECB and the Fed – as diverging interest-rate rules in such a two-country DSGE framework. Since the length of the time series for the Euro area and the U.S. available from a common data source is limited, such a small-scale approach seems particularly appealing. If diverging interest-rate rules were indeed a good approximation to the real behavior of the ECB and the Fed, a model allowing for these differences should be characterized by improved predictive ability compared to the standard case with identical interest-rate rules. This approach is not only well-founded on legal differences between the statutes of the ECB and the Fed, but is also corroborated by the data.

First, we run a stochastic simulation consisting of 11,000 draws with the two-country DSGE models yet to be introduced in Section 2 and calibrated as in Section 4 while employing the pure perturbation algorithm by Schmitt-Grohé and Uribe (2004). Discarding the first 1,000 draws as burn-in draws to minimize the impact of the starting values, we obtain simulated statistical moments implying that the DSGE model with diverging interest-rate rules delivers a lower volatility (standard deviation) of Euro area producer-price inflation (76.1% of the corresponding volatility of U.S. producer-price inflation) and Euro area consumer-price inflation (95.3%) relative to the U.S. at the expense of a higher volatility of the output gap (142.9%). Not surprisingly, the assumption of identical interest-rate rules for both central banks in turn delivers very similar simulated statistical moments.

Second, actual quarterly OECD and Eurostat data for the Euro area of 17 and the U.S. from 1996Q2 to 2011Q2 (see Figure 1 in Section 3) corroborate the findings of the simulated model with diverging interest-rate rules: a lower volatility of Euro area producer-price (60.2%) and consumer-price inflation (68.9%) relative to U.S. values, but almost the same volatility of the output gap (99.4%). Moreover, a lower volatility of consumer-price inflation and output growth in the Euro area relative to the U.S. has also been confirmed by Benati and Goodhart (2011, Figure 17) for annual data from 1999 to 2008.

In addition to other macroeconometric models, central banks in particular are typically interested in using their customized closed- and open-economy DSGE models for empirical policy analysis, forecasting, or both. While some authors, e.g. Smets and Wouters (2004) and Adolfson *et al.* (2007) for the Euro area, Smets and Wouters (2007) and Edge *et al.* (2010) for the U.S., find that their DSGE models are able to forecast well in comparison to (Bayesian) vector-autoregressive ((B)VAR) benchmarks, others obtain mixed results:

Rubaszek and Skrzypczyński (2008) find that both DSGE models and (B)VAR benchmarks are characterized by inferior forecasting accuracy compared to the Philadelphia Fed Survey of Professional Forecasters.

Lees *et al.* (2011), who employ the DSGE-VAR approach favored by Del Negro and Schorfheide (2004) and Del Negro *et al.* (2007) on data from New Zealand, find that a DSGE-VAR model is often outperformed by its BVAR competitor. Gupta and Kabundi (2010) find for the South African economy that large-scale BVAR models outperform DSGE, small-scale BVAR and dynamic factor models in most occasions. However, Gupta and Kabundi (2011) also find that both DSGE and (B)VAR models are typically outperformed by *large* factor models. Finally, Wang (2009) shows for U.S. data that, in the short run, a factor model outperforms a competing DSGE model while, in the long run, the theory-based DSGE model gains ground over the purely data-driven competitor.

Using GMM estimation techniques, Belke and Klose (2010) find evidence for significant differences in the signs of the parameters of extended Taylor (1993)-type interest-rate rules of the Fed before and after the beginning of the subprime crisis, but do not for the ECB. Where sign reversal of the Fed's reaction coefficient occurs (impact of consumer-price inflation and credit growth on the short-run nominal interest rate turns negative, impact of asset-price inflation turns positive), the ECB's reaction coefficients maintain their original signs in combination with an even higher overall significance.

Moreover, Sahuc and Smets (2008) find differences in the degree of central bank activism of the ECB and the Fed measured by the number of changes to their main refinancing rates (the Fed revised its short-run nominal interest rate more than twice as often as the ECB between 1999 and 2004) for two separately estimated DSGE models. These differences can largely be explained by differences in the size and type of structural shocks, however.

In general, Čihák *et al.* (2009) find that, during the first stage of the financial crisis, the ECB's monetary policy transmission mechanism continued to work, albeit with decreased efficiency. For the ECB to remain credible in the future in terms of fulfilling its mandate, of course, it will have to repeal any non-standard measures adopted during the crisis to ensure the smooth functioning of the monetary policy transmission mechanism, at the latest when consumer-price inflation persistently reaches levels above its goal of below but close to 2% over the medium term.

Consequently, we employ a small-scale two-country DSGE model of the Euro area and the U.S. that is characterized by diverging interest-rate rules. This small-scale two-country DSGE model is based on earlier research by Corsetti and Pesenti (2001), Obstfeld and Rogoff (2001), and Gunter (2009). The model's diverging rules reflect the differing mandates of the ECB and the Fed, respectively. Due to its primary objective of price stability, the ECB is charged with conducting monetary policy by considering producer-price inflation only, whereas the Fed is assumed to conduct its policy by taking into account the output gap in addition to producer-price inflation (dual mandate). Using quarterly OECD and Eurostat data from 1996Q2 to 2011Q2, we estimate the model with a standard calibration and Bayesian techniques and find posterior distributions of the model's structural parameters that are in line with the literature.

We evaluate the out-of-sample forecasting performance of this model for prediction horizons one to four in comparison to the same two-country DSGE model but with identical interest-rate rules as well as two BVAR benchmarks of lag order one and two. The BVAR benchmarks are chosen as competitors since they employ similar estimation techniques and are *ex ante* characterized by superior predictive ability in terms of log data density. The DSGE model with identical interest-rate rules, in turn, constitutes a natural competitor to the

DSGE model with diverging interest-rate rules. This will allow for *ceteris paribus* comparisons across model specifications to assess the empirical relevance of this specific friction – non-zero sensitivities on the output gap on the part of both central banks – to fit the data in terms of forecasting performance. A better forecasting performance of the less restricted DSGE model with diverging interest-rate rules relative to the DSGE model with identical interest-rate rules would corroborate our idea of the importance of allowing for the differences in the mandates of the ECB and the Fed.

Our main findings are as follows. In terms of the root mean squared error (RMSE) and the mean absolute error (MAE), the DSGE model with diverging interest-rate rules outperforms the DSGE model with identical interest-rate rules in almost 70% of all cases for almost all variables across forecast horizons (most prominent for one-quarter-ahead predictions), thereby corroborating the idea of employing diverging interest-rate rules. It also compares well with the BVAR benchmarks, especially for U.S. producer- and consumer-price inflation, and the terms of trade, as for the latter it attains the overall smallest RMSE and MAE for (almost) all horizons.

The good performance of the DSGE models relative to the BVAR benchmarks is partly due to the quarterly re-estimation of the models, which makes their free parameters quasi time-variant (see Giraitis et al. , 2014, for an overview) and, hence, the model structure itself overall more flexible towards capturing turning points in the business cycle, e.g. the onset of the financial crisis. However, for forecast horizons one and two we mostly find significantly better forecasting accuracy in terms of Harvey-Leybourne-Newbold statistics for the benefit of Bayesian benchmarks only. For forecast horizons three and four, the null hypothesis of equal forecasting accuracy can seldom be rejected across models.

The remainder of the article is structured as follows. Section 2 outlines the small-scale two-country DSGE model with diverging interest-rate rules, Section 3 describes the quarterly OECD and Eurostat data for the Euro area and the U.S., Section 4 presents the estimation approach and discusses the estimation results, and Section 5 introduces the benchmark models in comparison to which we assess the forecasting performance of the DSGE model under scrutiny. Finally, Section 6 concludes.

## 2 The model

The subsequent small-scale two-country DSGE model is based on earlier work by Corsetti and Pesenti (2001) and Obstfeld and Rogoff (2001) and corresponds to the one developed in Gunter (2009), which can be consulted for various derivations.<sup>1</sup>

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<sup>1</sup>Corsetti and Pesenti (2001) explore the international transmission mechanism and the welfare properties of different types of money-supply and government-spending shocks, whereas Obstfeld and Rogoff (2001) mostly concentrate on the issue of risk premia on nominal exchange rates finding that the exchange risk premium can be explicitly calculated as a function of underlying money-supply shocks. Gunter (2009) explores the reaction of the models' endogenous variables on simulated exogenous structural shocks in terms of impulse responses while extending the work of Corsetti and Pesenti (2001) and Obstfeld and Rogoff (2001) by assuming nominal rigidities in terms of Calvo (1983) pricing. Corsetti and Pesenti (2001) and Obstfeld and Rogoff (2001), in turn, assume one-period-in-advance nominal wage and price contracts, respectively.

## 2.1 Preferences, consumption and price indices

Let us assume that world population is constant over time and consists of a continuum of unit mass of infinitely lived atomistic households characterized by identical preferences. Let us assume further perfect information and rational expectations on the part of all agents. There are two countries, where domestic – henceforth: Euro area – households live on the segment  $[0, n]$  of the unit interval while foreign – henceforth: U.S. – households live on the remaining segment  $(n, 1]$ .

The discounted stream of expected period utilities of the representative Euro area household reads as follows:<sup>2</sup>

$$U_t = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left[ \frac{C_s^{1-\rho}}{1-\rho} + \frac{\chi}{1-\varepsilon} \left( \frac{M_s}{P_s} \right)^{1-\varepsilon} - \frac{\gamma}{1+\xi} L_s^{1+\xi} \right] \right\}. \quad (1)$$

The above utility function is a constant elasticity of substitution (CES) composite separable in its arguments real consumption  $C$ , real money balances  $M/P$  (where  $P$  denotes the domestic consumer price index (CPI), so-called money-in-the-utility-function model), and leisure  $-L$  such that the partial derivatives of the utility function with respect to one variable are independent of all other variables.  $\beta$  denotes an intertemporal discount factor ( $0 < \beta < 1$ ). Moreover, the following holds for the various parameters:  $\chi, \gamma, \xi > 0$  and  $0 < \rho, \varepsilon < 1$ .  $\rho$  is the coefficient of relative risk aversion in consumption or the inverse of the intertemporal elasticity of substitution of real consumption,  $\xi$  denotes the inverse of the elasticity of labor supply.

The utility function of the representative foreign household is the same as Eq. (1), except that  $C^*$  may differ from  $C$ , as well as  $M^*$  from  $M$ ,  $P^*$  from  $P$ ,  $\chi^*$  from  $\chi$ ,  $\gamma^*$  from  $\gamma$ , and  $L^*$  from  $L$ . Consequently, real and nominal U.S. variables are denoted by a superscript asterisk. In addition, nominal U.S. variables are denominated in U.S. dollars. This holds except for internationally traded bonds, where U.S. bond holdings indexed by a superscript asterisk are denominated in euros. Since (most) U.S. equations are completely analogous to Euro area equations, we restrict ourselves on the presentation of the latter.

The total Euro area consumption index  $C$  from Eq. (1) is defined as a population-weighted per-capita Cobb-Douglas composite of Euro area and U.S. commodity bundles, which implicitly makes the simplifying assumption that all consumption goods are tradable and that there are no trading costs:

$$C_t := \frac{C_{t,H}^n C_{t,F}^{1-n}}{n^n (1-n)^{1-n}}. \quad (2)$$

The commodity bundles  $C_H$  and  $C_F$  are CES composites of differentiated final goods produced in the Euro area ( $C_H$ ) or in the U.S. ( $C_F$ ) as in Dixit and Stiglitz (1977), thereby expressing households' love of variety:

$$C_{t,H} := \left[ \left( \frac{1}{n} \right)^{\frac{1}{\theta}} \int_0^n C_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \quad (3)$$

$$C_{t,F} := \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\theta}} \int_n^1 C_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}. \quad (4)$$

The optimal consumption-based Euro area CPI associated with Eq. (5) is a population-weighted Cobb-Douglas

<sup>2</sup>A possible superscript  $i$  to distinguish individual variables is suppressed throughout the analysis for the sake of better legibility.

composite of Euro area and U.S. producer price indices (PPI):

$$P_t = P_{t,H}^n P_{t,F}^{1-n}, \quad (5)$$

where these sub-indices are again optimal consumption-based CES composites of Euro area and U.S. final goods prices:

$$P_{t,H} = \left[ \frac{1}{n} \int_0^n P_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}, \quad (6)$$

$$P_{t,F} = \left[ \frac{1}{1-n} \int_n^1 P_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}. \quad (7)$$

For the sake of simplicity, we assume that the law of one price holds for consumers across all individual goods at all times:

$$P_t(z) = EXR_t P_t^*(z) \quad \forall z \in [0, 1], \quad (8)$$

where  $EXR$  denotes the endogenously determined nominal exchange rate in price quotation (Euros per U.S. dollar).

Thus, as Euro area and U.S. households are characterized by identical preferences, the law of one price implies that absolute purchasing power parity always holds for the CPI (5):

$$P_t = EXR_t P_t^*. \quad (9)$$

The demand functions of the representative Euro area household for individual Euro area  $C(h)$  and U.S. goods  $C(f)$  read as follows:

$$C_t(h) = \frac{1}{n} \left[ \frac{P_t(h)}{P_{t,H}} \right]^{-\theta} C_{t,H}, \quad (10)$$

$$C_t(f) = \frac{1}{1-n} \left[ \frac{P_t(f)}{P_{t,F}} \right]^{-\theta} C_{t,F}, \quad (11)$$

where  $z = h \in [0, n]$  denotes a typical differentiated good  $z$  produced in the Euro area and  $z' = f \in (n, 1]$  another typical differentiated good  $z'$  produced in the U.S.

Eq. (5) implies that the demand curves for the composite Euro area and U.S. goods,  $C_H$  and  $C_F$ , are given by:

$$C_{t,H} = n \left( \frac{P_{t,H}}{P_t} \right)^{-1} C_t, \quad (12)$$

$$C_{t,F} = (1-n) \left( \frac{P_{t,F}}{P_t} \right)^{-1} C_t. \quad (13)$$

Now we make use of the fact that world consumption  $C^w$  equals the population-weighted sum of total Euro area and total U.S. consumption, where  $C^w$  then denotes per capita as well as total world consumption since world population is normalized to 1:

$$C_t^w := nC_t + (1-n)C_t^*. \quad (14)$$



Combining Eq. (14) with Eqs. (8), (10), (11), (12), and (13) we finally obtain the *global* demand functions for individual Euro area and U.S. goods in terms of (total) world consumption:

$$C_t^w(h) = \left[ \frac{P_t(h)}{P_{t,H}} \right]^{-\theta} \left( \frac{P_{t,H}}{P_t} \right)^{-1} C_t^w, \quad (15)$$

$$C_t^w(f) = \left[ \frac{P_t(f)}{P_{t,F}} \right]^{-\theta} \left( \frac{P_{t,F}}{P_t} \right)^{-1} C_t^w. \quad (16)$$

## 2.2 Households

The representative Euro area household maximizes her objective function (1) subject to the following sequence of intertemporal budget constraints (in nominal terms) with respect to her decision variables  $C_t, M_t, B_t,$  and  $L_t$ :

$$W_t L_t + (1 + i_{t-1}) B_{t-1} + M_{t-1} + \Gamma_t(h) \geq P_t C_t + M_t + B_t + P_t \tau_t. \quad (17)$$

$W$  denotes the endogenously determined nominal wage being the remuneration for supplying labor, which is identical across households ( $L = L(h)$ ), on the assumed-to-be perfectly competitive labor market.  $i_{t-1}$  denotes the (short-run) nominal interest rate between period  $t - 1$  and period  $t$  on risk-free one-period non-government bonds  $B_{t-1}$  carried over from period  $t - 1$ . These nominal bonds are denominated in euros and are supposed to be internationally tradable.

Money holdings  $M_{t-1}$  can also be transferred from  $t - 1$  to  $t$ , but yield no nominal return. Consumption goods, however, are perishable and cannot be stored.  $\Gamma_t(h)$  are instantaneous profits of the representative household acting as a producer of an individual, differentiated Euro area good  $h$ , which will be explained in more detail below. Finally, let  $\tau$  denote non-distortionary real lump-sum taxes.<sup>3</sup>

Again, for the representative U.S. household the intertemporal budget constraint is very similar to Eq. (17). Since internationally traded bonds are supposed to be denominated in euros, U.S. bond holdings in denominated in euros  $B^*$ , however, first have to be divided by the nominal exchange rate before they enter the U.S. intertemporal budget constraint:  $B^*/EXR$ . Moreover,  $W^*$  may differ from  $W$ ,  $i^*$  from  $i$ ,  $\Gamma^*(f)$  from  $\Gamma(h)$ , as well as  $\tau^*$  from  $\tau$ . Hence, the sequence of U.S. intertemporal budget constraints (in nominal terms) reads as follows:

$$W_t^* L_t^* + (1 + i_{t-1}^*) \frac{B_{t-1}^*}{EXR_{t-1}} + M_{t-1}^* + \Gamma_t^*(f) \geq P_t^* C_t^* + M_t^* + \frac{B_t^*}{EXR_t} + P_t^* \tau_t^*. \quad (18)$$

Similar to Corsetti and Pesenti (2001), this equation implies that the realized nominal return on internationally traded bonds at the beginning of period  $t$  in U.S. dollars is given by:

$$(1 + i_{t-1}^*) = \frac{EXR_{t-1}}{EXR_t} (1 + i_{t-1}). \quad (19)$$

The maximization of the utility function (1) subject to the budget constraint (17) then holding with equality is undertaken by maximizing the corresponding Lagrangian and yields the subsequent first order conditions for a

<sup>3</sup>The government is assumed to set its expenditures equal to its revenues at all times such that its budget is always in balance and no seignorage can occur (see Obstfeld and Rogoff, 2001):  $M_t - M_{t-1} + P_t \tau_t = 0$ .

utility maximum:

$$\frac{C_t^{-\rho}}{P_t} = \beta(1 + i_t)E_t \left[ \frac{C_{t+1}^{-\rho}}{P_{t+1}} \right]. \quad (20)$$

This is the intertemporal Euler equation for real consumption stating that the marginal rate of substitution between real consumption in  $t$  and in  $t + 1$  equals their discounted relative prices.

Moreover, we obtain that in a utility maximum the marginal rate of substitution between real money balances and real consumption equals the opportunity costs of holding money:

$$\chi \frac{\left(\frac{M_t}{P_t}\right)^{-\varepsilon}}{C_t^{-\rho}} = \frac{i_t}{1 + i_t}. \quad (21)$$

Finally, we also get the subsequent labor supply equation:

$$\gamma \frac{L_t^\xi}{C_t^{-\rho}} = \frac{W_t}{P_t}, \quad (22)$$

which states that the marginal rate of substitution between labor and real consumption equals their relative prices, the real consumer wage.

### 2.3 Firms

Let us assume that agents in the Euro area and in the U.S. do not only act as utility maximizing households, but also as monopolistically competitive producers of final goods, which are producible without the input of intermediate goods. In contrast to their role as households whose preferences are assumed to be identical, all commodities are differentiated in order to satisfy the households' love of variety.

Individual Euro area output is produced according to the following linear production function:

$$Y_t(h) = A_t L_t(h). \quad (23)$$

Eq. (23) is a production function in labor only. For the sake of simplicity, physical capital is omitted as additional input factor throughout the analysis. This step can be justified by the short- to medium-run perspective of the model.  $A$  is a random variable denoting an exogenous aggregate productivity shock, which can be interpreted as a transitory process innovation.

Households need not be self-employed, but it is assumed that Euro area firms can employ Euro area workers only as well as U.S. firms shall be allowed to employ U.S. workers only.

Producers' instantaneous profits  $\Gamma_t(h)$  are given by:

$$\Gamma_t(h) = P_t(h)Y_t(h) - W_t L_t(h). \quad (24)$$

Relative to the producer's own price, Eq. (24) rearranges to:

$$\frac{\Gamma_t(h)}{P_t(h)} = Y_t(h) - \frac{W_t}{P_t(h)}L_t(h) = Y_t(h) - \frac{W_t}{P_t(h)}\frac{Y_t(h)}{A_t} = Y_t(h) - \kappa_t Y_t(h), \quad (25)$$

where we have made use of the production function (23). In Eq. (25)  $\kappa := W/[P(h)A]$  is defined as individual real marginal production cost.

For now, let us assume that all goods prices are flexible. Then each Euro area producer charges the same price denoted by the Euro area PPI ( $P_H = P(h)$ ). Thus, instantaneous profits rearrange to:

$$\Gamma_t(h) = P_{t,H}Y_t(h) - W_tL_t(h). \quad (26)$$

Maximizing Eq. (26) with respect to  $Y(h)$  and using the fact that in case of goods market clearing the output of a single producer equals global demand for the differentiated good ( $Y(h) = C^w(h)$ ), we get the standard first-order condition for a profit maximum in a model of monopolistic competition:

$$\frac{W_t}{P_{t,H}A_t} = \frac{\theta - 1}{\theta} := \kappa_t^{flex}, \quad (27)$$

which states that in a profit maximum associated with flexible prices, the corresponding real marginal production cost, which is defined as  $\kappa^{flex}$ , times the aggregate productivity shock  $A$  equals the real producer wage  $W/P_H$ .

## 2.4 Market clearing under flexible prices

Let us begin with the equilibrium conditions on the world markets for Euro area and U.S. goods denominated in euros:

$$P_{t,H}Y_t = P_tC_t^w, \quad (28)$$

$$P_{t,F}Y_t^* = P_tC_t^w, \quad (29)$$

where the left-hand side of Eq. (28) denotes global supply of and the right-hand side global demand for Euro area goods.<sup>4</sup>

Eqs. (28) and (29) immediately collapse to the definition of the terms of trade:

$$S_t := \frac{P_{t,F}}{P_{t,H}} = \frac{EXR_t P_{t,F}^*}{P_{t,H}} = \frac{Y_t}{Y_t^*}, \quad (30)$$

which is the ratio of imported goods' over exported goods' prices from the perspective of the Euro area or the ratio of Euro area output over U.S. output. Thus, a realization of  $S > 1$  is advantageous for Euro area output, whereas a realization of  $S < 1$  would be advantageous for U.S. output.

Using the domestic intertemporal budget constraint (17) plus further manipulations eventually yield the Euro

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<sup>4</sup>For reasons of brevity, we do not present the derivation of the equilibrium conditions on the money markets here (see Gunter, 2009, for more details).

area and U.S. balance of payment identities:

$$P_{t,H}Y_t - P_t C_t + i_{t-1}B_{t-1} \equiv B_t - B_{t-1}, \quad (31)$$

$$P_{t,F}Y_t^* - P_t C_t^* + i_{t-1}B_{t-1}^* \equiv B_t^* - B_{t-1}^* \quad (32)$$

with the left-hand side of Eq. (31) representing the Euro area's current account and the right-hand side its capital account.

Internationally tradable bonds are supposed to be in zero net world supply:

$$nB_t + (1 - n)B_t^* = 0. \quad (33)$$

Assuming that international bond holdings have initially been zero  $B_0 = B_0^* = 0$  together with Eqs. (14), (31), (32), and (33) implies that  $B_t = B_t^* = 0$  at all times according to Corsetti and Pesenti (2001) and Obstfeld and Rogoff (2001). Then Eqs. (31) and (32) simplify to the following:

$$C_t = \frac{P_{t,H}Y_t}{P_t}, \quad (34)$$

$$C_t^* = \frac{P_{t,F}Y_t^*}{P_t}. \quad (35)$$

Using the definition of the terms of trade (30) the preceding equations can be rewritten as:

$$C_t = S_t^{n-1}Y_t, \quad (36)$$

$$C_t^* = S_t^n Y_t^*. \quad (37)$$

These are the conditions for Euro area and U.S. goods market clearing, which imply that households across countries always consume exactly their real incomes (see Obstfeld and Rogoff, 2001).

Moreover,  $B_0 = B_0^* = 0$  together with Eqs. (14), (31), (32), and (33) also implies that  $C_t = C_t^* = C_t^w$  at all times such that

$$C_t = C_t^* = C_t^w = nC_t + (1 - n)C_t^* = nS_t^{n-1}Y_t + (1 - n)S_t^n Y_t^* = Y_t^n (Y_t^*)^{1-n}, \quad (38)$$

while making use of Eqs. (36) and (37).

In other words, Cobb-Douglas preferences for the Euro area and U.S. commodity bundles as in Eq. (2) together with producer-currency pricing and the absence of preference shocks imply under the assumption of completely flexible prices that any shock that reduces the supply of output of a country will increase its price in equal proportion. Thus, the value of its real income remains unchanged and the allocation under complete markets can be achieved without trade in bonds.

As a consequence, consumption shares across countries are not only time-constant but even equal as stated in Obstfeld and Rogoff (2001). Since current and capital accounts between the two countries are in balance at all times and in all possible states of the world, the mechanism of adjustment to shocks in the world economy will only be represented by evolution of the terms of trade, but not by changes in the countries' net asset positions.

These properties are consistent with the actual evolution of Euro area and U.S. current account data: the current

account earnings from as well as the current account expenditures to the respective other country in per cent of nominal Euro area and U.S. GDP have only reached values between 2% and 5% according to quarterly Eurostat data between 2003Q1 and 2011Q2. Thus, trade in goods and services between the two countries takes place to a non-negligible extent.

This fact notwithstanding, the current account *balance* between the two countries in per cent of nominal Euro area and U.S. GDP has been hovering around 0% throughout the sample and reached a time average of only 0.4% (0.3%) of nominal Euro area (U.S.) GDP. This finding corroborates the validity of the present model's properties of time-constant net asset positions.

By combining Eqs. (22), (27), and (36) with the CPI (5) we obtain two equations in  $W/P = (W/P_H)S^{n-1}$  which can be solved for  $L$ :

$$L_t = S_t^{\frac{(n-1)(1-\rho)}{\xi}} \left( \frac{A_t}{\gamma} \right)^{\frac{1}{\xi}} \left( \frac{\theta-1}{\theta} \right)^{\frac{1}{\xi}} Y_t^{-\frac{\rho}{\xi}}. \quad (39)$$

Eq. (39) states that in an equilibrium on the perfectly competitive labor market, Euro area employment positively depends on the aggregate productivity shock  $A$  and flexible-price real marginal production cost  $(\theta-1)/\theta$ , but negatively on the terms of trade  $S$  and Euro area output  $Y$ .

Combining Eq. (39) with the production function (23) and solving for  $Y$ , we finally obtain the Euro area flexible-price equilibrium output  $Y^{flex}$ :

$$Y_t^{flex} = S_t^{\frac{(n-1)(1-\rho)}{\xi+\rho}} A_t^{\frac{\xi+1}{\xi+\rho}} \left( \frac{\theta}{\theta-1} \right)^{-\frac{1}{\xi+\rho}} \gamma^{-\frac{1}{\xi+\rho}}. \quad (40)$$

According to Eq. (40), Euro area flexible-price equilibrium output positively depends on the aggregate productivity shock  $A$ , yet negatively on the terms of trade  $S$  and the flexible-price mark-up factor  $\theta/(\theta-1)$ .

## 2.5 Log-linear approximation and nominal rigidities

Since the DSGE model above cannot be solved in closed form, we have to log-linearize it around its non-stochastic zero-inflation steady state. Moreover, for money not to be neutral in the short run and monetary policy to be effective after all (see Clarida *et al.*, 1999), we need some form of nominal rigidities in addition to the assumption of monopolistic competition as in Dixit and Stiglitz (1977).

It is straightforward to derive the dynamic IS curves for both countries by log-linearizing the Euro area intertemporal Euler equation for real consumption (20) in combination with the Euro area goods markets clearing condition (36) as well as their U.S. analogues around the non-stochastic zero-inflation steady state.

Accordingly, we obtain:

$$\hat{y}_t = E_t[\hat{y}_{t+1}] + \frac{1}{\rho} \{E_t[\pi_{t+1}] - \hat{i}_t\} - (1-n)E_t[\Delta s_{t+1}], \quad (41)$$

$$\hat{y}_t^* = E_t[\hat{y}_{t+1}^*] + \frac{1}{\rho} \{E_t[\pi_{t+1}^*] - \hat{i}_t^*\} + nE_t[\Delta s_{t+1}]. \quad (42)$$

Except for all types of interest rates, lower-case Latin letters denote natural logarithms of the corresponding

variables and that all lower-case variables are given in percentage deviations from the non-stochastic zero-inflation steady state, which is denoted by hats. The zero-inflation steady-state values themselves are denoted by upper bars. Furthermore,  $\bar{i} = \bar{i}^* = (1 - \beta)/\beta$  holds for the zero-inflation steady-state nominal interest rates, both in the Euro area and in the U.S.

These two dynamic IS curves represent *aggregate demand* in both countries, where Eq. (41) can be interpreted as follows: current Euro area demand is higher than its zero-inflation steady-state value if the expected Euro area output deviation  $E_t[\hat{y}_{t+1}]$  is positive (interpretable as an expected peak in the business cycle). There is also a clear positive relation of current demand to expected CPI inflation  $E_t[\pi_{t+1}] := E_t[p_{t+1}] - p_t$  (households consume more today if prices are expected to increase in the future) and a negative relation to current deviations from the zero-inflation steady-state nominal interest rate  $\hat{i}_t$  (investing in nominal bonds is relatively attractive compared to buying consumption goods).

Moreover, there are also spill-over effects from the U.S., which affect current Euro area demand through the expected evolution of the terms of trade  $E_t[\Delta s_{t+1}]$ : current Euro area demand negatively depends on an expected increase in the latter since terms of trade expected to augment mean that imported goods from the U.S. become more expensive relative to Euro area goods.  $(1 - n)$  denotes the degree of openness of the Euro area to the U.S. Since the Euro area degree of openness coincides with the size of the U.S. due the definition of the CPI (5) as a population-weighted Cobb-Douglas composite, there is no home bias in consumption.

The New Keynesian Philips curves (NKPCs) for both countries can be derived by log-linearizing the price-setting equations of Euro area firms as well as their U.S. analogue around the non-stochastic zero-inflation steady-state.

We introduce nominal rigidities in terms of sticky prices by assuming Calvo (1983) contracts on the part of firms. Calvo (1983) contracts imply that each producer is only allowed to reset her price with probability  $(1 - \delta)$  in any given period, independent of the time since the last adjustment. Therefore, a measure of  $(1 - \delta)$  of firms reset their prices each period, while a measure of  $\delta$  of firms keep their prices constant and simply adjust their individual output in order to meet demand.  $1/(1 - \delta)$  then captures the average duration of a price:

$$\pi_{t,H} = \beta E_t[\pi_{t+1,H}] + \frac{(1 - \delta)(1 - \delta\beta)}{\delta} \hat{\kappa}_t, \quad (43)$$

$$\pi_{t,F}^* = \beta E_t[\pi_{t+1,F}^*] + \frac{(1 - \delta^*)(1 - \delta^*\beta)}{\delta^*} \hat{\kappa}_t^*. \quad (44)$$

In Eq. (43),  $\pi_{t,H} := p_{t,H} - p_{t-1,H}$  is defined as current Euro area PPI inflation, which typically differs from CPI inflation in an open economy. The NKPC (43) states that current Euro area PPI inflation  $\pi_{t,H}$  is an increasing function of both expected Euro area PPI inflation  $E_t[\pi_{t+1,H}]$  and the deviation of current Euro area real marginal production cost from its zero-inflation steady-state value  $\hat{\kappa}_t := \kappa_t - \kappa_t^{flex}$ .

Furthermore, let us assume that *setting a new price at home* and *setting a new price abroad* are stochastically independent events. As Euro area and U.S. firms both set their prices in the currency of the countries where they are located, the present model features producer currency pricing as in Clarida *et al.* (2002).

Nonetheless, we want to express Eqs. (41), (42), (43), and (44) in terms of the output gap, which shall be defined as the difference between actual and flexible-price output deviations:  $x_t := \hat{y}_t - \hat{y}_t^{flex}$  and  $x_t^* := \hat{y}_t^* - (\hat{y}_t^*)^{flex}$ . In

order to rewrite Eqs. (43) and (44) in terms of  $x$  and  $x^*$ , respectively, we have to take a closer look at the ratio of the sticky-price real marginal production cost  $\kappa_t$  and its flexible-price counterpart  $\kappa_t^{flex}$  as given by Eqs. (25) and (27):

$$\frac{\kappa_t}{\kappa_t^{flex}} = \frac{\theta W_t S_t^{1-n}}{(\theta - 1) P_t A_t}. \quad (45)$$

Combining Eq. (45) with the labor supply curve (22), the production function (23), and the condition for Euro area goods market clearing (36), we obtain after some manipulation:

$$\frac{\kappa_t}{\kappa_t^{flex}} = \left( \frac{Y_t}{Y_t^{flex}} \right)^{\xi + \rho}, \quad (46)$$

where  $Y_t^{flex}$  denotes the domestic flexible-price equilibrium output as given by equation (40). Log-linearizing this expression around the non-stochastic zero-inflation steady-state yields:

$$\hat{\kappa}_t = (\xi + \rho)(\hat{y}_t - \hat{y}_t^{flex}) = (\xi + \rho)x_t. \quad (47)$$

Hence, by using Eq. (47), Eqs. (41), (42), (43), and (44) rearrange to:

$$x_t = E_t[x_{t+1}] + \frac{1}{\rho} \{E_t[\pi_{t+1}] - \hat{i}_t\} - (1 - n)E_t[\Delta s_{t+1}] + E_t[\hat{y}_{t+1}^{flex}] - \hat{y}_t^{flex}, \quad (48)$$

$$x_t^* = E_t[x_{t+1}^*] + \frac{1}{\rho} \{E_t[\pi_{t+1}^*] - \hat{i}_t^*\} + nE_t[\Delta s_{t+1}] + E_t[(\hat{y}_{t+1}^*)^{flex}] - (\hat{y}_t^*)^{flex}, \quad (49)$$

$$\pi_{t,H} = \beta E_t[\pi_{t+1,H}] + \frac{(1 - \delta)(1 - \delta\beta)(\xi + \rho)}{\delta} x_t + u_t, \quad (50)$$

$$\pi_{t,F}^* = \beta E_t[\pi_{t+1,F}^*] + \frac{(1 - \delta^*)(1 - \delta^*\beta)(\xi + \rho)}{\delta^*} x_t^* + u_t^*. \quad (51)$$

$u_t$  denotes an exogenously given, stationary AR(1) process of the form  $u_t = \zeta_u u_{t-1} + \eta_{u,t}$  ( $0 < \zeta_u < 1$ ) with the exogenous error term  $\eta_u$  assumed to be i.i.d.  $\sim N(0, \sigma_{\eta_u}^2)$ . This AR(1) process can be interpreted as a transitory cost-push shock reflecting determinants of real marginal production cost which do not move proportionally with the output gap (see Clarida *et al.*, 2001).

The two NKPCs represent *aggregate supply* in both countries and are isomorphic to their closed-economy counterparts, where Eq. (50) can be interpreted as follows: the positive short-run *trade-off* between current Euro area PPI inflation  $\pi_{t,H}$  and the current Euro area output gap  $x_t$  can be seen. However, this is not really a trade-off to be exploited by policymakers since  $\pi_{t,H}$  is also positively related to (discounted) expected Euro area PPI inflation  $\beta E_t[\pi_{t+1,H}]$ .

It will turn out to be convenient that the following holds for  $E_t[\hat{y}_{t+1}^{flex}] - \hat{y}_t^{flex}$  when we make use of the log-linear version of the current Euro area flexible-price equilibrium output according to Eq. (40) and its expected counterpart:

$$E_t[\hat{y}_{t+1}^{flex}] - \hat{y}_t^{flex} = \frac{(n - 1)(1 - \rho)}{\xi + \rho} E_t[\Delta s_{t+1}] + \frac{\xi + 1}{\xi + \rho} E_t[\Delta a_{t+1}], \quad (52)$$

where the transitory productivity shock  $a_t$  is assumed to follow an exogenously given, stationary AR(1) process of the form  $a_t = \zeta_a a_{t-1} + \eta_{a,t}$  ( $0 < \zeta_a < 1$ ) with the exogenous error term  $\eta_a$  assumed to be i.i.d.  $\sim N(0, \sigma_{\eta_a}^2)$ .

In consequence, the dynamic IS curves (48) and (49) rearrange to:

$$x_t = E_t[x_{t+1}] + \frac{1}{\rho}\{E_t[\pi_{t+1}] - \hat{l}_t\} + \frac{(n-1)(\xi+1)}{\xi+\rho}E_t[\Delta s_{t+1}] + \frac{\xi+1}{\xi+\rho}E_t[\Delta a_{t+1}], \quad (53)$$

$$x_t^* = E_t[x_{t+1}^*] + \frac{1}{\rho}\{E_t[\pi_{t+1}^*] - \hat{l}_t^*\} + \frac{n(\xi+1)}{\xi+\rho}E_t[\Delta s_{t+1}] + \frac{\xi+1}{\xi+\rho}E_t[\Delta a_{t+1}^*]. \quad (54)$$

Since CPI and PPI inflation typically differ in open economy models (see, e.g., Clarida *et al.*, 2001), we need two equations linking these two types of inflation rates. In doing so, we use the log-linear version of the Euro area CPI definition (5) and its U.S. analogue:

$$\pi_t = \pi_{t,H} - (n-1)\Delta s_t + e_t, \quad (55)$$

$$\pi_t^* = \pi_{t,F}^* - n\Delta s_t + e_t^*, \quad (56)$$

where  $e_t$  is assumed to follow an exogenously given, stationary AR(1) process of the form  $e_t = \zeta_e e_{t-1} + \eta_{e,t}$  ( $0 < \zeta_e < 1$ ) with the exogenous error term  $\eta_e$  assumed to be i.i.d.  $\sim N(0, \sigma_{\eta_e}^2)$ . Since in reality Euro area and U.S. CPIs do not only consist of Euro area and U.S. goods prices, the error term  $e_t$  can be interpreted as a wedge between the present definition (55) and the realized CPIs.

## 2.6 Diverging interest-rate rules

As mentioned in Section 1, we model the differences in monetary policy mandates and objectives of the ECB and the Fed in terms of diverging Taylor (1993)-type interest-rate rules with feedback of (some of) the endogenous variables. The feedback is introduced to circumvent price level (and inflation) indeterminacy as shown by Sargent and Wallace (1975), which would be associated with purely exogenous interest-rate targets (see Woodford, 2003, pp. 101–106, for Neo-Wicksellian cashless and money-in-the-utility-function models such as the one used in the present article as given with Eq. (1)).

Consequently, the interest-rate rules differ to the extent that the Fed is supposed to conduct its monetary policy by considering current U.S. PPI inflation  $\pi_{t,F}^*$  and the current US output gap  $x_t^*$  (dual mandate), while the ECB imposes its monetary policy by taking into account current EU PPI inflation  $\pi_{t,H}$  *only* (primary objective). This difference is due to the fact that all conceivable policy goals of the ECB besides price stability can be interpreted as secondary:

$$\hat{l}_t = \alpha\pi_{t,H} + \omega\hat{l}_{t-1} + v_t, \quad (57)$$

$$\hat{l}_t^* = \alpha^*\pi_{t,F}^* + \iota^*x_t^* + \omega^*\hat{l}_{t-1}^* + v_t^*. \quad (58)$$

The ECB's interest rate rule (57) can be described as follows:  $\alpha$  ( $\alpha > 0$ ) denotes the sensitivity of the ECB to current Euro area PPI inflation  $\pi_{t,H}$ .<sup>5</sup> In addition, the rule incorporates some degree of inertia of the monetary policy instrument  $i$  itself, which is measured by the parameter  $\omega$  ( $0 < \omega < 1$ ). The parameter  $1 - \omega$ , in

<sup>5</sup>A standard result of optimal non-cooperative monetary policy under discretion in a two-country DSGE framework featuring producer currency pricing is that central banks should target current PPI inflation instead of CPI inflation (see Clarida *et al.*, 2002). Moreover, Galí and Monacelli (2005) show within a small-open-economy DSGE framework with producer currency pricing that for the majority of cases welfare losses in terms of units of steady-state consumption are lower for a Taylor (1993)-type interest-rate rule sensitive to PPI inflation compared to a rule sensitive to CPI inflation or to a peg of the nominal exchange rate.



turn, measures the degree of adjustment to the zero-inflation steady-state value of the nominal interest rate  $\bar{i} = (1 - \beta)/\beta$ , which could also be interpreted as interest-rate target. The feature of interest-rate inertia is rather an empirical finding than an implication of the mandates of the central banks (see Woodford , 2003, pp. 95–96).

$\iota^*$  ( $\iota^* > 0$ ) in Eq. (58) denotes the sensitivity of the Fed to the current U.S. output gap  $x_t^*$ , where  $\iota = 0$  is assumed to hold for the ECB. As noted by Del Negro *et al.* (2007), setting one model parameter equal to zero represents a reduction by one of the many nominal and real frictions imposed by a DSGE model on the data. This will allow for *ceteris paribus* comparisons across model specifications in Sections 1 and 5 in order to assess the empirical relevance of a friction  $\iota \neq 0$  to fit the data in terms of forecasting performance. Since the signs of the elasticities of the central banks' policy instruments to endogenous variables are all positive so that they react counter-cyclically to their changes, the policies can also be characterized as having a *lean-against-the-wind* property (see Clarida *et al.* , 1999).

The Taylor principle, which states that the monetary authority ought to react to an increase in current PPI inflation by augmenting its policy instrument *more* than one for one in order to allow for a determinate rational expectations equilibrium (see Woodford , 2003, p. 40), is assumed to be fulfilled by both central banks ( $\alpha, \alpha^* > 1$ ) later on in Section 4.<sup>6</sup>

Moreover, in Eq. (57),  $v_t$  denotes an exogenously given, stationary AR(1) process of the form  $v_t = \zeta_v v_{t-1} + \eta_{v,t}$  ( $0 < \zeta_v < 1$ ) with the exogenous error term  $\eta_v$  assumed to be i.i.d.  $\sim N(0, \sigma_{\eta_v}^2)$ . This AR(1) process can be interpreted as a transitory monetary policy shock, whereby a positive realization of  $\eta_v$  denotes a contractionary shock.

Finally, we need an equation expressing the terms of trade as a function of the remaining endogenous variables. Let us use the log-linear version of Eq. (19), which reads  $\hat{i}_{t-1} = \Delta \ln EXR_t + \hat{i}_{t-1}^*$ , in order to substitute for  $\Delta \ln EXR_t$  in the log-linear representation of current evolution of the terms of trade from Eq. (30):  $\Delta s_t = \Delta \ln EXR_t + \pi_{t,F}^* - \pi_{t,H}$ . Hence, we obtain:

$$\Delta s_t = \hat{i}_{t-1} - \hat{i}_{t-1}^* + \pi_{t,F}^* - \pi_{t,H} + d_t, \quad (59)$$

where  $d_t$  is assumed to follow an exogenously given, stationary AR(1) process of the form  $d_t = \zeta_d d_{t-1} + \eta_{d,t}$  ( $0 < \zeta_d < 1$ ) with the exogenous error term  $\eta_d$  assumed to be i.i.d.  $\sim N(0, \sigma_{\eta_d}^2)$ . Since in reality condition (19) may not always hold with equality, the error term  $d_t$  can be interpreted as a wedge between the present definition (59) and the realized evolution of the terms of trade.

In summary, with Eqs. (50), (51), (53), (54), (55), (56), (57), (58), and (59) we have derived a determined system of nine expectational difference equations in nine endogenous variables, which can now be taken to the data. Henceforth, we will refer to the two-country DSGE model with diverging interest-rate rules as *DSGE-DIV* ( $\iota = 0$ ). The model with identical interest-rate rules ( $\iota \neq 0$ ), in turn, will be referred to as *DSGE-SAME*.

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<sup>6</sup>The Taylor principle in its purest form is not a necessary condition for equilibrium determinacy for an interest-rate rule of type (58). Instead, the condition  $(1 - \delta^*)(1 - \delta^*\beta)(\xi + \rho)/\delta^*(\alpha^* - 1) + (1 - \beta)\iota^* > 0$  would be a necessary and sufficient condition for equilibrium determinacy in case of a contemporaneous interest-rate rule (see Bullard and Mitra , 2002).

### 3 The data

We use revised quarterly data for the Euro area of 17 and the U.S. ranging from 1996Q2 to 2011Q2. The data are taken from Eurostat (output gaps) and OECD (all other variables) and are vintages as retrieved on November 10, 2011. This gives us 61 observations for the full sample. For historical realizations of the variables see Figure 1.<sup>7</sup>

[Figure 1 about here.]

The sample starts in 1996Q2 because no earlier observations are available for CPI inflation for the Euro area of 17. We use OECD and Eurostat data since these are reliable and publicly accessible data sources and all variables are defined in a comparable manner for the two economies of interest. Moreover, we concentrate on the Euro area of 17 since this is the Euro area as it existed until the end of 2013.

According to Smets and Wouters (2005), the convergence process within the (future) Euro area may – at the earliest – have started in the mid-1980s. Given the fall of the Iron Curtain no earlier than 1989/1990 and the beginning of the transition of the centrally planned economies in Central and Eastern Europe to market economies in its aftermath, the start of the convergence process within the (future) Euro area should safely be assumed no sooner than the signature of the Treaty on the European Union (the so-called Maastricht Treaty) in 1992, which includes the Maastricht convergence criteria for entering the third stage of the Economic and Monetary Union with the view to finally adopting the euro as a common currency.

The output gaps ( $x, x^*$ ) are modeled as the natural logarithm of seasonally and working-day adjusted real GDP minus potential output. Potential output is proxied by the trend of log real GDP as obtained from the Hodrick-Prescott filter (see Hodrick and Prescott, 1997, penalty parameter  $\lambda = 1,600$  for quarterly data) over the whole sample (1996Q2–2011Q2) since we employ revised data for all variables and are interested in evaluating ex-post forecasting accuracy only. This step constitutes a deviation from the definition of the output gap as given in Section 2 and is mainly due to practical considerations (non-availability of consistently defined data for flexible-price-equilibrium output for the Euro area and the U.S.). However, Orphanides and van Norden (2005) confirm that the Hodrick-Prescott filter (among other univariate and multivariate measures) is a useful output-gap estimate as long as revised data are considered.

The output gaps of both economies turned strongly negative in the course of the financial crisis ( $x$  in 2009Q1,  $x^*$  in 2008Q4) and show signs of a slight recovery no earlier than 2011Q1 ( $x$ ) and in 2010Q2 ( $x^*$ ), respectively. Besides the run-up to the financial crisis and the crisis itself, the data also cover the build-up of the new economy bubble in the late 1990s and its burst in the early 2000s.

PPI inflation rates ( $\pi_H, \pi_F^*$ ) are modeled as the quarter-on-quarter change in per cent divided by 100 of the index of total producer prices (domestically produced goods sold at home and abroad) in manufacturing. We restrict ourselves to this index since the present model assumes that firms employ producer currency pricing and that only final goods are produced and traded.

CPI inflation rates ( $\pi, \pi^*$ ) are modeled as the quarter-on-quarter change in per cent divided by 100 of the

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<sup>7</sup>Summary statistics of the variables are available on request.

Harmonized Index of Consumer Prices (HICP) in case of the Euro area of 17 and the U.S. consumer price index in case of the U.S., respectively. We choose to use CPI inflation instead of the GDP deflator since we explicitly focus on prices for domestic and imported goods faced by households.

At the beginning of the financial crisis ( $\pi_H$ : 2008Q4–2009Q2;  $\pi_F^*$ ,  $\pi$ ,  $\pi^*$ : 2008Q4–2009Q1) a short deflationary period in producer and consumer prices can be observed, whereby the decrease of U.S. price indices is more severe than of their Euro area equivalents. Over the whole sample, PPI inflation in both economies is more than twice as volatile as CPI inflation and both U.S. inflation measures are generally more volatile than their Euro area counterparts.

The short-run nominal (interbank) interest rates ( $\hat{i}$ ,  $\hat{i}^*$ ) are modeled as the 3-month Euro Interbank Offered Rate (EURIBOR) in case of the Euro area (ECB synthetic rates calculated using national rates, LIBOR where available, weighted by GDP prior to 1999Q1) and the rate on 3-month nationally traded certificates of deposit (CDs) issued by commercial banks in case of the U.S., respectively, in per cent per annum divided by 400 minus 0.01 (approximate quarterly zero-inflation steady-date nominal/real interest rate for  $\beta = 0.99$ ). A value of 0.01 also roughly corresponds to the *realized* average values for both economies' short-run quarterly nominal interest rates between 1996Q2 and 2011Q2. The below-average values of  $\hat{i}$  (since 2009Q1) and  $\hat{i}^*$  (since 2008Q1) in response to the lower main refinancing rates of the ECB and the Fed have remained in that range until the end of the sample in 2011Q2.<sup>8</sup>

Finally, we calculate the terms of trade ( $\Delta s$ ) ourselves by using the first difference of the natural logarithm of the 3-month average of the nominal exchange rate of euros (of European Currency Units, ECUs, prior to 1999Q1) per U.S. dollar plus  $\pi_F^*$  minus  $\pi_H$ :  $\Delta s_t = \Delta \ln EXR_t + \pi_{t,F}^* - \pi_{t,H}$ .

The terms of trade feature the highest volatility of all macroeconomic variables not only due to the impact of PPI inflation, but also due to the volatile nature of a floating nominal exchange rate. The advantageousness of the terms of trade for Euro area output (positive realization) and U.S. output (negative value) therefore also changes frequently.

## 4 Estimation

### 4.1 Estimation approach and prior distributions

Estimation of the two-country DSGE model *DSGE-DIV* is carried out by employing Bayesian techniques (see, e.g., An and Schorfheide, 2007, for a survey on Bayesian inference in DSGE models). As laid out, e.g., in Lütkepohl (2005, pp. 222–223), for Bayesian estimation it is assumed that non-sample information on a generic parameter vector  $\psi$  available prior to estimation is summarized in its prior probability density function (PDF)  $g(\psi)$ . The sample information on  $\psi$ , however, is summarized in its sample PDF given by  $f(\mathbf{y}|\psi)$ , which is algebraically identical to the likelihood function  $l(\psi|\mathbf{y})$ . By reweighting the likelihood function by

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<sup>8</sup>We are aware of the fact that the ECB did not operate before January 1, 1999. This means that the time series used for the Euro area short-run nominal interest rate during the period from 1996Q2 until 1998Q4 is a synthetic rate. A rejection of this approximation would considerably reduce the already quite limited sample. Furthermore, in the case of the U.S., a short-run nominal interest rate with a 3-month maturity had to be chosen, thereby precluding the use of the Federal Funds Rate, the maturity of which is overnight.

an informative prior the so-called *dilemma of absurd parameter estimates* (see An and Schorfheide , 2007) can be circumvented, which would otherwise result in probably unrealistic posterior means. That is why pure maximum likelihood estimation is not as prominent in DSGE estimation as Bayesian inference.

The distribution of the parameter vector  $\psi$  conditional on the sample information contained in  $\mathbf{y}$  can be summarized by  $g(\psi|\mathbf{y})$ , which is known as posterior PDF. The posterior distribution, which contains all information available for the parameter vector  $\psi$ , is proportional to the likelihood function times the prior PDF.

Since the posterior distribution cannot be determined analytically, we have to adopt some type of Monte Carlo Markov Chain sampling algorithm to simulate the distribution of the parameter vector  $\psi$  (see, e.g., Christoffel *et al.* , 2008). In particular, we adopt the Metropolis-Hastings algorithm, whose steps are outlined, e.g., in Koop (2003, pp. 92–94), consisting of two parallel Monte Carlo Markov Chains with 250,000 draws all together. Before computing the posterior mean and covariance, 20% of the draws are discarded as burn-in draws to mitigate the impact of the starting values. All computations are performed with the DYNARE preprocessor for MATLAB, which can be downloaded for free from [www.dynare.org](http://www.dynare.org).

For the present framework, the parameters introduced in Section 2 constitute the parameter vector  $\psi := (n, \beta, \rho, \xi, \delta, \delta^*, \alpha, \alpha^*, \iota^*, \omega, \omega^*, \zeta_a, \zeta_a^*, \zeta_u, \zeta_u^*, \zeta_e, \zeta_e^*, \zeta_v, \zeta_v^*, \zeta_d, \sigma_{\eta_a}, \sigma_{\eta_a^*}, \sigma_{\eta_u}, \sigma_{\eta_u^*}, \sigma_{\eta_e}, \sigma_{\eta_e^*}, \sigma_{\eta_v}, \sigma_{\eta_v^*}, \sigma_{\eta_d})'$ , which includes the eleven structural parameters of which nine are estimated, as well as the autoregressive parameters ( $\zeta$ ) and standard errors ( $\sigma_{\eta}$ ) of the nine AR(1) disturbances.

Similar to Smets and Wouters (2005) and Sahuc and Smets (2008), who estimate separate models, we employ the same priors for both countries. The calibration of the parameter vector  $\psi$  (prior means, standard deviations and PDFs) is standard and largely follows Smets and Wouters (2007). It can be obtained from Table 1 together with the estimation results (posterior distributions) of *DSGE-DIV*. Deviations from the Smets and Wouters (2007) calibration include  $\delta^{(*)} = 0.75$  (prior mean only), which is taken from Smets and Wouters (2003) and Rubaszek and Skrzypczyński (2008) implying a somewhat longer a-priori average duration of a price of  $1/(1 - \delta^{(*)}) = 1/(1 - 0.75) = 4$  quarters. The assumed Gamma prior PDFs for  $\alpha^{(*)}$  and  $\iota^*$  also deviate from Smets and Wouters (2007) and follow Lees *et al.* (2011) because only positive values are plausible if the aforementioned countercyclical *lean-against-the-wind* policy stance is assumed (see Clarida *et al.* , 1999). Moreover, the prior mean of  $\iota^* = 0.5$  corresponds to the original Taylor (1993) value. We employ the same calibration for  $\iota \neq 0$  as for  $\iota^*$  to formulate *DSGE-SAME* (prior mean, prior standard deviation, as well as prior distribution), e.g. to assess this model's forecasting performance relative to *DSGE-DIV* as done in Section 5. We assume little prior knowledge about the standard errors of the AR(1) disturbances and therefore assign Inverse Gamma prior PDFs.

The country size  $n = 0.5$  is kept fixed throughout estimation because it is no generically economic parameter and set to 0.5 because the Euro area and the U.S. are approximately equal-sized countries as measured by both GDP and population. The intertemporal discount factor  $\beta = 0.99$  is also kept fixed because it is often only weakly identified. A standard value of 0.99 implies an approximate quarterly zero-inflation steady-date nominal/real interest rate of  $(1 - \beta)/\beta = 0.01$ . Altogether, very similar calibrations can be found in other empirical DSGE papers on the Euro area and the U.S. such as the ones already cited in Section 1: Smets and Wouters (2003, 2005); Adolfson *et al.* (2007); Christoffel *et al.* (2008); Rubaszek and Skrzypczyński (2008); Edge *et al.* (2010). This short list does not claim to be exhaustive.

The chosen calibration ensures that the Blanchard and Kahn (1980) conditions (7 eigenvalues larger than 1 in modulus for 7 forward-looking variables) are satisfied such that there is a unique stationary solution to the determined system of expectational difference equations (50), (51), (53), (54), (55), (56), (57), (58), and (59), which render the rational expectations equilibrium determinate. Moreover, the identification toolbox for DSGE models that is incorporated in DYNARE (see Ratto , 2011, for a description) gives us additional confidence in the goodness of the chosen calibration. Its Monte Carlo option, which uses information about the whole prior distribution, suggests that all model parameters are identified not only at their prior means as indicated in Table 1, but also over a loop of 250 random parameter draws over the entire prior distribution.

## 4.2 Estimation results

Generally speaking, the parameter estimates are in line with DSGE parameter estimates obtained by other authors for the Euro area and the U.S. Nonetheless, it is worthwhile to discuss the estimation results in case there are differences to parameter estimates obtained elsewhere in the literature.

The two parameters stemming from household utility (1), the inverse of the intertemporal elasticity of substitution of real consumption  $\rho$  and the inverse of the elasticity of labor supply  $\xi$ , are the same across countries as identical preferences are assumed. Concerning  $\rho$ , the posterior mean of 2.0258 is somewhat higher than the values obtained separately for the Euro area (1.391, Smets and Wouters , 2003) and the U.S. (1.38, Smets and Wouters , 2007). Smets and Wouters (2005) obtain median estimates of 1.13 (Euro area) and 1.95 (U.S., which is closer to our result), whereas Sahuc and Smets (2008) obtain more similar values of 1.231 for the Euro area and 1.282 for the U.S. By contrast, Rubaszek and Skrzypczyński (2008) obtain a value for  $\rho$  of only 0.97 for the U.S.

The posterior mean of  $\xi$  of 2.9789 is also slightly higher than 2.503 (see Smets and Wouters , 2003) obtained for the Euro area or 1.83 (see Smets and Wouters , 2007) for the U.S, implying that U.S. labor supply is more elastic than Euro area labor supply. Nonetheless, we also obtain a lower posterior mean of  $\rho$  compared to  $\xi$  indicating that intertemporal substitution of real consumption on a quarterly basis is more elastic than labor supply, which is plausible. Rubaszek and Skrzypczyński (2008) again obtain a similar posterior mean for U.S. data: 1.97. Also Smets and Wouters (2005) ( $\xi = 2.00$ ;  $\xi^* = 2.88$ ) and Sahuc and Smets (2008) ( $\xi = 2.204$ ;  $\xi^* = 2.361$ ) obtain posterior medians that are within the same range as ours, but with U.S. labor supply being more elastic.

At 0.4085, the posterior mean of the Euro area degree of price stickiness  $\delta$  is higher than the posterior mean of  $\delta^*$  (0.3764), whereby both values imply an average duration of a price below but close to two quarters. With 0.905 (see Smets and Wouters , 2003) for the Euro area and 0.66 (see Smets and Wouters , 2007) for the U.S., also other authors obtain higher values for the Euro area than in U.S. For instance, Adolfson *et al.* (2007) obtain a value of 0.883 for domestic prices in the Euro area (posterior mode), whereas Rubaszek and Skrzypczyński (2008) obtain a value of 0.78 for the U.S. Christoffel *et al.* (2008) find an even higher posterior mean of 0.921 for Euro area domestic prices, which is in the neighborhood of the posterior medians found by Smets and Wouters (2005) and Sahuc and Smets (2008) for both the Euro area and the U.S.

The posterior mean of the ECB's sensitivity to PPI inflation  $\alpha = 1.9011$  is higher than  $\alpha^* = 1.7714$  suggesting a more hawkish stance on inflation on the part of the ECB. Other authors obtain similar values such as 1.688 (see

Smets and Wouters , 2003), 1.710 (see Adolfson *et al.* , 2007) (posterior mode), or 1.900 (see Christoffel *et al.* , 2008) for the Euro area or 2.04 (see Smets and Wouters , 2007), 1.73 (see Rubaszek and Skrzypczyński , 2008) for the U.S. Also, the posterior mean of the Fed's sensitivity to the output gap  $\iota^*$ , at 0.0775, is in line with the literature. Smets and Wouters (2007) obtain, with a posterior mean of 0.08, a similarly low sensitivity of the Fed to the output (gap), which is close to zero. For the model *DSGE-SAME*, with  $\iota \neq 0$ , we obtain an estimate of  $\iota = 0.0653$ , which is somewhat lower than the posterior mean of  $\iota^* = 0.0713$  obtained in that case.

Interestingly, Smets and Wouters (2005) ( $\alpha = 1.41; \alpha^* = 1.49; \iota = 0.11; \iota^* = 0.09$ ) and Sahuc and Smets (2008) ( $\alpha = 1.529; \alpha^* = 1.831; \iota = 0.071; \iota^* = 0.064$ ) also obtain posterior medians that are in the neighborhood of our estimates for the central banks' sensitivities on inflation and output (gap), but in both cases the ECB seems to have a more dovish stance on monetary policy than the Fed. The reason for this difference may be that the samples of these studies (1983Q1–2002Q2 and 1985Q1–2004Q4, respectively) barely overlap with our period under study.<sup>9</sup>

Interest-rate smoothing as measured by  $\omega, \omega^*$  is of importance for both central banks. With 0.8493 and 0.8594, respectively, the posterior means are in line with 0.956 (see Smets and Wouters , 2003), 0.867 (see Christoffel *et al.* , 2008), 0.874 (see Adolfson *et al.* , 2007) (posterior mode) for the ECB and 0.81 (see Smets and Wouters , 2007), 0.76 (see Rubaszek and Skrzypczyński , 2008) for the Fed. Also Smets and Wouters (2005) and Sahuc and Smets (2008) obtain similar results.

Similar to Smets and Wouters (2007), aggregate productivity ( $\zeta_a = 0.9767; \zeta_a^* = 0.9669$ ) and cost-push ( $\zeta_u = 0.9889; \zeta_u^* = 0.9845$ ) shocks are more persistent than monetary policy shocks ( $\zeta_v = 0.2831; \zeta_v^* = 0.2160$ ). However, the reason why monetary policy shocks turn out to be less persistent than other shocks is the fact that the lagged nominal interest rate has already been included in the interest-rate rules (interest-rate smoothing). Also the shocks representing the wedges between the model definition and the realization of the CPIs and the evolution of the terms of trade feature only a low degree of persistence ( $\zeta_e = 0.0200; \zeta_e^* = 0.0306; \zeta_d = 0.0237$ ).

[Table 1 about here.]

Since the modes of the posterior distributions and the posterior modes do not deviate much from each other, we have used a sufficient number of draws for the Metropolis-Hastings algorithm. Similar to Smets and Wouters (2007), the generally lower variance of the posterior distributions of the model parameters relative to the prior distributions (see Table 1) indicates that the data is informative on the model parameters. Moreover, desirable acceptance rates of candidate draws according to Roberts *et al.* (1997) are met across Monte Carlo Markov Chains (0.2802 and 0.2751, respectively).<sup>10</sup>

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<sup>9</sup>One caveat concerning the interpretation of the estimation results, which however is in line with the empirical findings from the literature, still needs to be addressed: the posterior means of both central banks' sensitivities to the output gap obtained in this study are very similar in size and close to zero. Therefore, from an econometric point of view, any quantitative statement in terms of one central bank putting more emphasis on the output gap than the other has to be taken with a grain of salt.

<sup>10</sup>The convergence of parameter estimates can also be deemed fulfilled in terms of the univariate convergence diagnostics by Brooks and Gelman (1998), for which the corresponding graphs are available on request. Graphs plotting prior and posterior distributions, as well as the posterior modes are also available on request.

## 5 Forecasting performance

### 5.1 Rival forecasting models

As noted by Smets and Wouters (2007) or Rubaszek and Skrzypczyński (2008), unconstrained VARs that are estimated using ordinary least squares or pure maximum likelihood are often overparameterized and may therefore perform poorly in forecasting. This finding and a similar estimation methodology makes Bayesian VARs (BVARs) natural atheoretical benchmarks to DSGE models. In doing so, we employ the Sims (2003) variant of the so-called Minnesota or Litterman prior for BVAR estimation and forecasting, which is incorporated in DYNARE. The Minnesota prior is an informative prior developed by and specified in Doan *et al.* (1984) on an otherwise unconstrained VAR with intercept, which imposes restrictions on the longer lags of a VAR rather than eliminating them (see, e.g., Gupta and Kabundi, 2011, for more details).

We follow Smets and Wouters (2007) concerning the calibration of the various prior parameters: the decay parameter is set to 1.0 (so-called linear decay), the overall tightness to 10 (representing a comparatively loose prior on own lags), the parameter determining the weight on the sum of coefficients or own-persistence to 2.0, and the parameter determining the weight on the co-persistence is set to 5.0. For the purpose of out-of-sample forecasting, we draw 10,000 random samples from the posterior distribution.

Table 2 presents the posterior predictive ability of candidate rival forecasting models in terms of the models' log data density, which is obtained from the modified harmonic mean estimator as in Geweke (1999) in case of DSGE models. As can be seen, *DSGE-DIV* attains a higher value than *DSGE-SAME*. Moreover, we will employ *BVAR(1)* and *BVAR(2)* as atheoretical time-series benchmarks since log data density decays as the lag order increases beyond 2.<sup>11</sup>

Calculating Bayes factors as in An and Schorfheide (2007) to assess the ex-ante posterior predictive ability of *DSGE-DIV* and the three remaining rival models *DSGE-SAME*, *BVAR(1)* and *BVAR(2)*, we conclude that *DSGE-DIV* is expected to slightly outperform *DSGE-SAME* (Bayes factor of  $e^{16}$ ), thus corroborating our idea of a model with diverging interest-rate rules being able to better capture the real behavior of the ECB and the Fed. However, both BVAR benchmarks are expected, based on Jeffreys (1961, p. 432), to (almost) *decisively* outperform both *DSGE-DIV* and *DSGE-SAME* (see Robert *et al.*, 2009). This may illustrate the flexible nature of the BVAR structure being able to capture turning points in the business cycle, e.g., the onset of the financial crisis, more easily than the more rigid DSGE structure with its time-constant parameters.

[Table 2 about here.]

After employing Bayesian model comparison to determine the four final rival forecasting models, we evaluate their ex-post pseudo-out-of-sample forecasting performance by conventional measures of forecasting accuracy based on dynamic forecasts of the nine endogenous variables in terms of the predictive mean for forecast horizons  $h = 1, \dots, 4$  while using expanding windows. This means that each model is re-estimated on a quarterly basis, while starting from the sub-sample 1996Q2–2006Q1 (40 observations) and expanding the estimation

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<sup>11</sup>This deterioration continues for *BVAR(5)* and *BVAR(6)*, which are not shown here. Using CPI instead of PPI inflation rates in the interest-rate rules would result in a deterioration of posterior predictive ability, which corroborates the importance of using PPI inflation rates in a two-country DSGE framework that features producer currency pricing.

window up to sub-samples 1996Q2–2011Q1 (60 observations for  $h = 1$ ), 1996Q2–2010Q3 (59 observations for  $h = 2$ ), 1996Q2–2010Q3 (58 observations for  $h = 3$ ), and 1996Q2–2010Q2 (57 observations for  $h = 4$ ), respectively. Altogether, this delivers  $T_1 = 21$  (for  $h = 1$ ),  $T_2 = 20$  (for  $h = 2$ ),  $T_3 = 19$  (for  $h = 3$ ), and  $T_4 = 18$  (for  $h = 4$ ) counterfactual observations. Due to the computational burden associated with full-fledged Metropolis-Hastings iterations, we pursue the quarterly re-estimation of the two DSGE models with one Monte Carlo Markov Chain consisting of 125,000 draws only, where 20% are discarded as burn-in draws.

Using the expanding windows (or recursive) forecasting technique also corresponds to a “natural” practitioner’s situation, where all information available up to the forecast origin is used for forecasting. Thus, we implicitly allow for the financial crisis from the second half of 2007 onward as the sub-samples for the first estimations associated with the forecasting evaluation exercise already end prior to 2007Q3. Subsequent estimation windows, however, include the crisis period.

## 5.2 Measures of forecasting accuracy

As measures of forecasting accuracy we employ the traditional root mean squared error (RMSE, see Table 3) and mean absolute error (MAE, see Table 4), whereby the latter is more sensitive to small deviations from zero, but less sensitive to large deviations since it is not computed based on squared losses (see Chatfield, 2001, p. 150). In general, a better forecasting accuracy of the DSGE models relative to the BVAR models would justify the constraints in the DSGE model as imposed by economic theory relative to the unconstrained BVAR specification (see Rubaszek and Skrzypczyński, 2008). Similar reasoning holds when we compare the performance of the less restricted *DSGE-DIV* with the performance of *DSGE-SAME*.

In Tables 3 and 4, cells that are shaded in gray denote the smallest errors among DSGE models, while values in boldface denote the smallest overall errors among rival forecasting models. In general terms, Euro area and U.S. PPI inflation, as well as the terms of trade are more difficult to predict across models and forecast horizons, which underlines the volatile nature of producer prices and nominal exchange rates. At forecast horizon  $h = 1$ , *DSGE-DIV* delivers more accurate results across variables in terms of both RMSE and MAE for all variables among DSGE models. When we increase the forecast horizon to  $h = 2, \dots, 4$ , *DSGE-SAME* is gaining ground at the expense of *DSGE-DIV*, especially when predicting the output gaps, Euro area CPI and PPI inflation, as well as the short-run U.S. nominal interest rate. However, altogether *DSGE-DIV* produces smaller forecasting errors than *DSGE-SAME* in 24 (RMSE) and 25 (MAE) out of 36 cases each, respectively.

*BVAR(2)*, in turn, often delivers the most accurate forecasts in terms of RMSE (14 cases) and MAE (18 cases) among all rival models, followed by *BVAR(1)* (RMSE: 10 cases, MAE: 12 cases), thereby corroborating the comparably good forecasting performance of Bayesian vector-autoregressions in the forecasting literature.

Nonetheless, the DSGE models often deliver the smallest overall RMSE for Euro area and U.S. PPI inflation, U.S. CPI inflation, and the terms of trade. Among these, *DSGE-DIV* (10 cases) is characterized by the smallest overall RMSE for U.S. PPI inflation (for  $h = 1, 2, 3, 4$ ), the terms of trade (for  $h = 1, 2, 4$ ), U.S. CPI inflation (for  $h = 1, 2$ ), and Euro area PPI inflation (for  $h = 2$ ) and lies therefore in level with *BVAR(1)*. *DSGE-SAME*, however, is only able to produce the smallest overall RMSE in 2 cases: Euro area PPI inflation (for  $h = 3$ ) and the terms of trade (for  $h = 3$ ). Pertaining to the MAE, the DSGE models (6 cases, *DSGE-DIV* only) are only



able to deliver the overall smallest values for a narrower range of variables and a smaller number of forecast horizons: Euro area (for  $h = 3$ ) and U.S. (for  $h = 2$ ) CPI inflation, and the terms of trade (for  $h = 1, 2, 3, 4$ ). In 18 cases *BVAR(2)* is the most accurate model, in 12 cases *BVAR(1)*.

One puzzling result of this forecasting evaluation exercise remains to be addressed, namely the ostensible contradiction that the DSGE models perform so well *in practice* (see Tables 3 and 4) despite being characterized *ex ante* by a relatively mediocre posterior predictive ability (see Table 2). The solution to this puzzle is the quarterly re-estimation of the models, which makes their free parameters quasi time-variant. Consequently, both DSGE specifications are gaining ground in terms of flexibility towards capturing turning points in the business cycle, e.g., the onset of the financial crisis (see Giraitis et al. , 2014, for an overview).

[Table 3 about here.]

[Table 4 about here.]

Finally, since the traditional measures of forecasting accuracy such as RMSE or MAE do not indicate whether a particular model such as *DSGE-DIV* significantly outperforms or underperforms its competitors, we have to consult the Harvey-Leybourne-Newbold (HLN) statistic on equal predictive accuracy developed by Harvey *et al.* (1997), which corrects the original Diebold-Mariano statistic on equal predictive accuracy (see Diebold and Mariano , 1995) for small samples. The test statistic is t-distributed with  $T - 1$  degrees of freedom under the null hypothesis of equal forecasting accuracy of two rival models.

In line with Rubaszek and Skrzypczyński (2008), the long-run variance in the denominator of the HLN test statistic is estimated in line with the Newey and West (1987) procedure using a modified Bartlett kernel, where the truncation lag is dependent on the number of observations, as proposed by Newey and West (1994). For calculating the HLN test statistic, we use squared loss differentials between two rival models in the numerator. A negative sign of the values in Table 5 indicates a smaller loss differential of *DSGE-DIV*, a positive sign a smaller loss differential of the respective rival model. (\*\*) denotes significance of the HLN statistic at the 5%, (\*) at the 10% level.

As noted by Wang (2009), in the presence of nested models (such as *DSGE-DIV* and *DSGE-SAME*) the HLN statistic has a non-standard asymptotic distribution and tests for equal predictive forecasting accuracy tuned to nested models such as the one suggested by Clark and McCracken (2001) should be preferred in principle. However, Wang (2009) also refers to Giacomini and White (2006) who derive that Diebold-Mariano type test statistics are still asymptotically standard-normally distributed for nested models when rolling estimation windows are employed (Theorem 4). The same authors note that this reasoning also holds for expanding estimation windows (see Giacomini and White , 2006) so that the HLN statistic on equal predictive accuracy can still be employed for the present choice of rival forecasting models.

[Table 5 about here.]

As we can infer from Table 5, the null hypothesis of equal forecast accuracy cannot be rejected at the 1% significance level in any case. However, we see some rejection of the null hypothesis of equal forecast accuracy at the 5% and 10% significance levels, especially for shorter forecast horizons. With only 13 out of 108 total cases, the number of cases with significant differences of forecasting accuracy between rival models

is comparably low. Although *DSGE-DIV* is characterized by smaller RMSE and MAE values compared to *DSGE-SAME* across variables and (almost all) forecast horizons, this outperformance is – apart from one case (U.S. PPI inflation for  $h = 2$ ) – not statistically significant. As indicated by many overall lowest RMSE and MAE values, the BVAR benchmarks (10 cases altogether) are jointly characterized by a significantly better forecasting performance than *DSGE-DIV* for the Euro area (for  $h = 4$ ) and the U.S. output gaps (for  $h = 1$ ), Euro area CPI inflation (for  $h = 2$ ), and the U.S. short-run nominal interest rate (for  $h = 1, 2$ ). Nonetheless, *DSGE-DIV* retains its good forecasting accuracy for the terms of trade as observed from the RMSE and MAE given in Tables 3 and 4 also in terms of the HLN statistic for  $h = 2, 3$  relative to *BVAR(1)*. Altogether, the null hypothesis of equal forecast accuracy can seldom be rejected for forecast horizons  $h = 3, 4$ .

The reason why the four rival forecasting models can only rarely outperform one another *significantly*, most likely, is the use of the expanding windows forecasting technique: based on Monte Carlo simulations, Pesaran and Pick (2011) find that averaging forecasts over different estimation windows almost always leads to a lower RMSE relative to forecasts that are based on rolling estimation windows, even in the presence of structural breaks. The authors confirm this general result by an application to financial data.

## 6 Conclusion

The main findings of this article can be summarized as follows. In terms of the RMSE and the MAE, the DSGE model with diverging interest-rate rules outperforms the DSGE model with identical interest-rate rules in almost 70% of all cases for almost all variables across forecast horizons, whereby the improvements in forecasting accuracy are most prominent for one-quarter-ahead predictions. It also compares well with the two BVAR benchmarks of lag order 1 and 2, especially for U.S. producer- and consumer-price inflation, and the terms of trade, as for the latter it attains the overall smallest RMSE and MAE for (almost) all horizons.

To a certain extent this improvement relative to ex-ante predictive ability is due to the quarterly re-estimation of the DSGE models, which makes their free parameters quasi time-variant and, hence, the model structure itself overall more flexible towards capturing turning points in the business cycle, e.g. the onset of the financial crisis. However, for forecast horizons one and two we mostly find significantly better forecasting accuracy in terms of HLN statistics for the benefit of Bayesian benchmarks only. For forecast horizons three and four, the null hypothesis of equal forecasting accuracy can seldom be rejected across models. The reason why the four rival forecasting models can only rarely outperform one another *significantly*, most likely, is the use of the expanding windows forecasting technique.

The overall picture of our analysis is that allowing for diverging interest-rate rules in DSGE forecasting is worthwhile for the following reasons. First, the DSGE model with diverging interest-rate rules attains lower RMSE and MAE values across variables and forecast horizons compared to the DSGE model with identical interest-rate rules, thus corroborating the importance of the differences in the mandates of the ECB and the Fed. Second, the model also compares well with the BVAR benchmarks, especially for U.S. producer- and consumer-price inflation, and the terms of trade. This is worth emphasizing since BVAR benchmarks frequently forecast better than DSGE models as can often be seen in the literature. Third, to the best of our knowledge, we are among the first to address the issue of diverging interest-rate rules within a two-country DSGE framework of

the Euro area and U.S. economies, which is one of the key contributions of this paper.

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Table 1: Prior and posterior distributions of the structural parameters of *DSGE-DIV*.

Parameter	Prior distribution			Posterior distribution				
	PDF	Mean	Std. Dev.	Mode	Std. Dev.	Mean	90% Conf. Int.	
$\rho$	Normal	1.5	0.37	2.0317	0.2333	2.0258	1.6138	2.4309
$\xi$	Normal	2.0	0.75	2.9385	0.6699	2.9789	1.8700	4.0339
$\delta$	Beta	0.75	0.1	0.4171	0.0399	0.4085	0.3425	0.4760
$\delta^*$	Beta	0.75	0.1	0.3828	0.0374	0.3764	0.3156	0.4397
$\alpha$	Gamma	1.5	0.25	1.8570	0.2234	1.9011	1.5216	2.2713
$\alpha^*$	Gamma	1.5	0.25	1.6891	0.2324	1.7714	1.3772	2.1661
$\iota^*$	Gamma	0.5	0.2	0.0607	0.0270	0.0775	0.0264	0.1240
$\omega$	Beta	0.75	0.1	0.8685	0.0677	0.8493	0.7500	0.9567
$\omega^*$	Beta	0.75	0.1	0.8772	0.0635	0.8594	0.7630	0.9576
$\zeta_a$	Beta	0.5	0.2	0.9772	0.0082	0.9767	0.9643	0.9906
$\zeta_a^*$	Beta	0.5	0.2	0.9685	0.0109	0.9669	0.9495	0.9895
$\zeta_u$	Beta	0.5	0.2	0.9931	0.0046	0.9889	0.9795	0.9983
$\zeta_u^*$	Beta	0.5	0.2	0.9900	0.0068	0.9845	0.9718	0.9975
$\zeta_e$	Beta	0.5	0.2	0.0174	0.0106	0.0200	0.0038	0.0350
$\zeta_e^*$	Beta	0.5	0.2	0.0260	0.0145	0.0306	0.0069	0.0523
$\zeta_v$	Beta	0.5	0.2	0.2890	0.0350	0.2831	0.2255	0.3435
$\zeta_v^*$	Beta	0.5	0.2	0.2187	0.0290	0.2160	0.1671	0.2650
$\zeta_d$	Beta	0.5	0.2	0.0195	0.0126	0.0237	0.0039	0.0414
$\sigma_{\eta_a}$	Inv. Gamma	0.1	$+\infty$	0.0347	0.0117	0.0411	0.0199	0.0624
$\sigma_{\eta_a^*}$	Inv. Gamma	0.1	$+\infty$	0.0360	0.0114	0.0435	0.0194	0.0701
$\sigma_{\eta_u}$	Inv. Gamma	0.1	$+\infty$	0.0235	0.0044	0.0256	0.0172	0.0335
$\sigma_{\eta_u^*}$	Inv. Gamma	0.1	$+\infty$	0.0308	0.0057	0.0331	0.0227	0.0427
$\sigma_{\eta_e}$	Inv. Gamma	0.1	$+\infty$	0.0231	0.0020	0.0235	0.0200	0.0270
$\sigma_{\eta_e^*}$	Inv. Gamma	0.1	$+\infty$	0.0257	0.0023	0.0262	0.0224	0.0301
$\sigma_{\eta_v}$	Inv. Gamma	0.1	$+\infty$	0.0195	0.0025	0.0204	0.0161	0.0246
$\sigma_{\eta_v^*}$	Inv. Gamma	0.1	$+\infty$	0.0301	0.0046	0.0322	0.0241	0.0402
$\sigma_{\eta_d}$	Inv. Gamma	0.1	$+\infty$	0.0442	0.0039	0.0450	0.0384	0.0513

*Note:* The calibration primarily follows Smets and Wouters (2007). Posterior results are obtained from estimating over the full sample (1996Q2–2011Q2). We employ MATLAB's `fmincon` optimization routine to retrieve the posterior modes.

Table 2: Posterior predictive ability of candidate rival forecasting models (log marginal data density).

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DSGE-DIV	1,737	BVAR(1)	1,835
DSGE-SAME	1,721	BVAR(2)	1,842
		BVAR(3)	1,819
		BVAR(4)	1,817

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*Note:* The log data densities of candidate rival forecasting models that are not ultimately used are given in gray.



Table 3: Root mean squared errors of rival forecasting models.

Horizon	Model	$x$	$x^*$	$\pi_H$	$\pi_F^*$	$\pi$	$\pi^*$	$\hat{i}$	$\hat{i}^*$	$\Delta s$
$h = 1$	DSGE-DIV	0.00949	0.01060	0.01379	<b>0.02517</b>	0.01060	<b>0.01085</b>	0.00231	0.00344	<b>0.03617</b>
	DSGE-SAME	0.00974	0.01133	0.01464	0.02582	0.01136	0.01249	0.00313	0.00373	0.04083
	BVAR(1)	0.00766	0.00746	0.01335	0.02701	0.01030	0.01176	0.00106	<b>0.00133</b>	0.04370
	BVAR(2)	<b>0.00671</b>	<b>0.00713</b>	<b>0.01209</b>	0.02629	<b>0.00879</b>	0.01174	<b>0.00100</b>	0.00135	0.04304
$h = 2$	DSGE-DIV	0.01545	0.01549	<b>0.01590</b>	<b>0.03053</b>	0.01035	<b>0.01210</b>	0.00370	0.00539	<b>0.03975</b>
	DSGE-SAME	0.01444	0.01528	0.01638	0.03180	0.01099	0.01403	0.00507	0.00543	0.04270
	BVAR(1)	0.01321	<b>0.01244</b>	0.01657	0.03389	<b>0.00555</b>	0.01414	0.00225	<b>0.00254</b>	0.06461
	BVAR(2)	<b>0.01294</b>	0.01288	0.01592	0.03215	0.00582	0.01300	<b>0.00224</b>	0.00278	0.05358
$h = 3$	DSGE-DIV	0.01994	0.01826	0.01615	<b>0.02996</b>	0.01042	0.01335	0.00470	0.00646	0.04419
	DSGE-SAME	0.01831	0.01776	<b>0.01599</b>	0.03013	0.01019	0.01380	0.00594	0.00604	<b>0.04286</b>
	BVAR(1)	0.01725	<b>0.01525</b>	0.01655	0.03378	0.01022	0.01277	0.00335	<b>0.00341</b>	0.07129
	BVAR(2)	<b>0.01719</b>	0.01615	0.01646	0.03157	<b>0.00951</b>	<b>0.01135</b>	<b>0.00332</b>	0.00378	0.05589
$h = 4$	DSGE-DIV	0.02407	0.02068	0.01667	<b>0.02910</b>	0.00984	0.01226	0.00544	0.00722	<b>0.04391</b>
	DSGE-SAME	0.02178	0.02001	0.01635	0.02936	0.00994	0.01371	0.00654	0.00646	0.04503
	BVAR(1)	0.02018	<b>0.01628</b>	<b>0.01574</b>	0.03073	<b>0.00653</b>	0.01139	0.00414	<b>0.00394</b>	0.06392
	BVAR(2)	<b>0.01988</b>	0.01727	0.01677	0.02946	0.00673	<b>0.01052</b>	<b>0.00401</b>	0.00432	0.05280

Note: Cells that are shaded in gray denote the smallest RMSE among DSGE models, while values in boldface denote the smallest RMSE among all rival forecasting models.

Table 4: Mean absolute errors of rival forecasting models.

Horizon	Model	$x$	$x^*$	$\pi_H$	$\pi_F^*$	$\pi$	$\pi^*$	$\hat{i}$	$\hat{i}^*$	$\Delta_s$
$h = 1$	DSGE-DIV	0.00651	0.00817	0.01073	0.01861	0.00882	0.00827	0.00151	0.00263	<b>0.02860</b>
	DSGE-SAME	0.00749	0.00878	0.01104	0.01879	0.00965	0.00985	0.00246	0.00269	0.03289
	BVAR(1)	0.00510	0.00550	0.00923	0.01796	0.00942	<b>0.00823</b>	<b>0.00068</b>	0.00106	0.03280
	BVAR(2)	<b>0.00427</b>	<b>0.00518</b>	<b>0.00815</b>	<b>0.01750</b>	<b>0.00805</b>	0.00870	0.00069	<b>0.00103</b>	0.03236
$h = 2$	DSGE-DIV	0.01156	0.01254	0.01246	0.02210	0.00920	<b>0.00874</b>	0.00263	0.00408	<b>0.03042</b>
	DSGE-SAME	0.01204	0.01178	0.01273	0.02405	0.00951	0.01107	0.00385	0.00381	0.03243
	BVAR(1)	0.00970	0.00938	<b>0.01077</b>	0.02116	<b>0.00413</b>	0.00988	0.00145	<b>0.00215</b>	0.04752
	BVAR(2)	<b>0.00859</b>	<b>0.00924</b>	0.01111	<b>0.02112</b>	0.00451	0.00876	<b>0.00139</b>	0.00220	0.04036
$h = 3$	DSGE-DIV	0.01649	0.01514	0.01311	0.02446	<b>0.00817</b>	0.01053	0.00341	0.00482	<b>0.03367</b>
	DSGE-SAME	0.01539	0.01408	0.01313	0.02445	0.00849	0.01098	0.00472	0.00442	0.03532
	BVAR(1)	0.01336	<b>0.01187</b>	<b>0.01113</b>	0.02236	0.00912	0.00887	0.00213	<b>0.00270</b>	0.05334
	BVAR(2)	<b>0.01258</b>	0.01203	0.01121	<b>0.02158</b>	0.00835	<b>0.00820</b>	<b>0.00206</b>	0.00288	0.04046
$h = 4$	DSGE-DIV	0.02039	0.01757	0.01348	0.02283	0.00828	0.00992	0.00424	0.00556	<b>0.03519</b>
	DSGE-SAME	0.01755	0.01561	0.01331	0.02344	0.00774	0.01069	0.00557	0.00530	0.03610
	BVAR(1)	0.01607	<b>0.01359</b>	<b>0.01117</b>	0.02196	<b>0.00515</b>	0.00824	0.00278	<b>0.00325</b>	0.04646
	BVAR(2)	<b>0.01559</b>	0.01402	0.01244	<b>0.02178</b>	0.00553	<b>0.00726</b>	<b>0.00270</b>	0.00341	0.03655

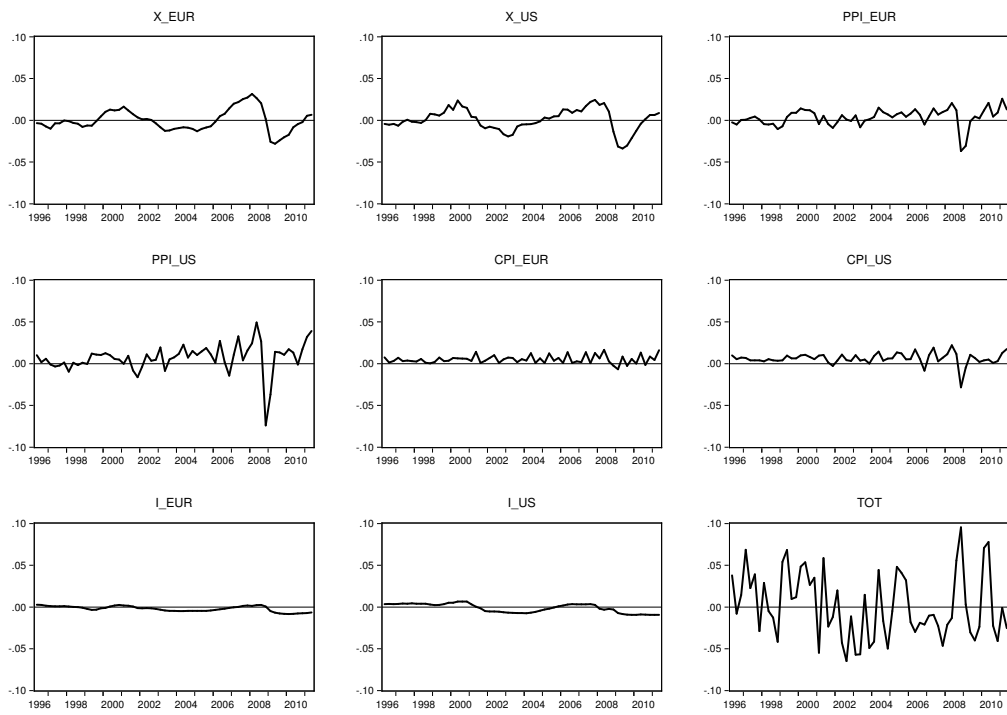
Note: Cells that are shaded in gray denote the smallest MAE among DSGE models, while values in boldface denote the smallest MAE among all rival forecasting models.

Table 5: Harvey-Leybourne-Newbold (HLN) statistics of rival forecasting models relative to *DSGE-DIV*.

Horizon	DSGE-DIV vs.	$x$	$x^*$	$\pi_H$	$\pi_F^*$	$\pi$	$\pi^*$	$\hat{i}$	$\hat{i}^*$	$\Delta_s$
$h = 1$	DSGE-SAME	-0.2669	-0.6928	-1.1362	-0.8909	-0.4791	-1.7086	-1.1429	-0.3384	-1.0695
	BVAR(1)	1.4479	<b>2.0624*</b>	0.3779	-0.5667	0.1772	-0.7233	1.5376	<b>2.0938**</b>	-0.9528
	BVAR(2)	1.4150	<b>1.8366*</b>	1.2707	-0.4090	1.0715	-0.7647	1.5249	<b>2.1322**</b>	-0.9191
$h = 2$	DSGE-SAME	0.4734	0.1755	-1.2375	<b>-2.2650**</b>	-0.5958	-1.2147	-1.1428	-0.0334	-0.6118
	BVAR(1)	1.3259	<b>1.7504*</b>	-0.2800	-0.7618	<b>2.1446**</b>	-1.0276	1.5369	<b>1.9011*</b>	<b>-1.7749*</b>
	BVAR(2)	1.4989	1.4632	-0.0121	-0.4844	<b>2.1322**</b>	-0.7272	1.4742	<b>1.8670*</b>	-1.4997
$h = 3$	DSGE-SAME	0.7589	0.2995	0.5172	-0.3984	0.1758	-0.1439	-1.1062	0.3030	0.3273
	BVAR(1)	1.3780	1.3127	-0.1465	-0.6837	0.0971	0.4841	1.3864	1.6416	<b>-1.7887*</b>
	BVAR(2)	1.8332	1.0330	-0.1222	-0.4511	0.4154	1.0396	1.3666	1.5369	-1.4481
$h = 4$	DSGE-SAME	1.1263	0.2433	0.8643	-0.5496	-0.0652	-0.5429	-1.0420	0.5392	-0.3609
	BVAR(1)	1.6329	1.1545	0.4561	-0.3652	1.2323	0.6235	1.3020	1.4240	-1.3564
	BVAR(2)	<b>1.8741*</b>	1.0899	-0.0492	-0.1224	1.1698	1.2638	1.4164	1.3905	-1.0125

*Note:* A negative sign of an HLN value indicates a smaller squared loss differential of *DSGE-DIV*, a positive sign a smaller loss differential of the respective rival model. (\*\*) denotes significance of the HLN statistic at the 5%, (\*) at the 10% level.

Figure 1: Historical realizations of variables.



Source: Eurostat and OECD (1996Q2-2011Q2).